MET 4501 Spring 2025: Practice Exam 2 Name: SOLUTION

Section 1: Multiple Choice (2 points each, 20 points total)

1. Which of the following characteristics are NOT associated with an ideal spring material? Select all that apply.
   - ○ High ultimate strength
   - ○ High yield strength
   - ● High modulus of elasticity
2. Which of the fatigue failure criteria listed below is the most conservative?
   - ● Goodman
   - ○ Gerber
   - ○ Morrow
3. Fatigue failure is characterized by:
   - ○ A sudden failure after a dynamic shock pulse.
   - ○ A gradual degradation in material properties due to high temperatures.
   - ● A failure after a number of load cycles under fluctuating stress.
   - ○ A failure due to yielding under a single static load.
4. When analyzing a beam subjected to pure torsion, which of the following stresses must be evaluated?
   - ○ Normal stress
   - ● Shear stress
   - ○ Bending stress
   - ○ Hoop stress
5. Fatigue failure typically occurs under:
   - ○ Static loading conditions
   - ● Repeated or fluctuating loads
   - ○ Uniform loading conditions
   - ○ Only when the stress exceeds the yield strength
6. Of the three major types of fatigue analysis discussed, which is most applicable for low-stress, high-cycle applications?
   - ● Stress-life method
   - ○ Strain-life method
   - ○ Linear elastic fracture mechanics
7. What is the endurance limit of a material?
   - ○ The maximum stress a material can withstand before permanent deformation occurs.
   - ● The magnitude of a completely reversed stress cycle that can be applied to the material indefinitely without failure, theoretically.
   - ○ The stress at which a material will begin to creep over time.
   - ○ The minimum stress required to initiate crack propagation in a material.

8. Estimate S_e' for an AISI 1018 HR steel. (See table below).
   - ○ 180 MPa
   - ● 200 MPa
   - ○ 220 MPa
   - ○ 240 MPa

$S_e' = 0.5 S_{ut}$

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

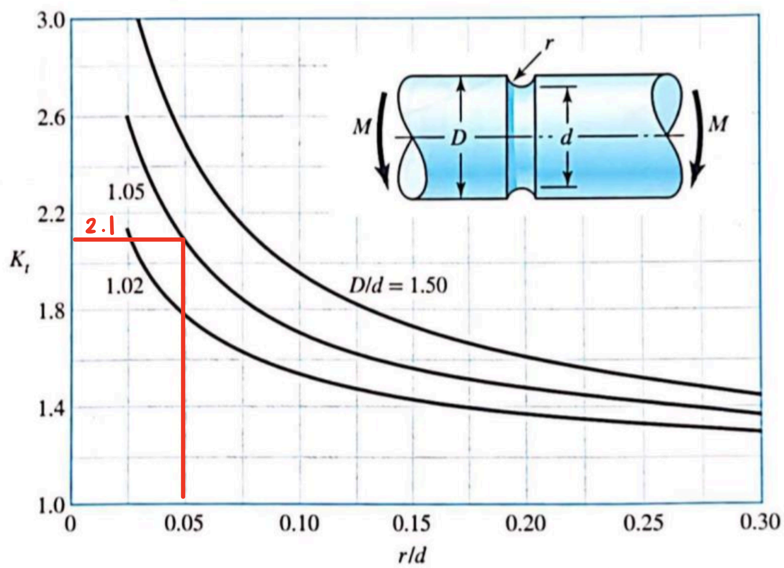
9. Which of the following are failure modes of a typical steel key? Select all that apply.
   - ● The key is sheared across its width
   - ● The key is crushed by the force from the keyseat/keyway.
   - ○ The key is elongated by high torque loads leading to high tensile stresses.
   - ○ The key melts due to excessive friction during normal operation.
10. Which of the following best describes the role of shot peening in spring design?
    - ○ It increases the spring stiffness by altering the wire diameter
    - ● It introduces compressive residual stresses to improve fatigue resistance
    - ○ It hardens the surface of the spring to resist corrosion
    - ○ It roughens the surface to improve the coefficient of friction.

Section 2: Problem-Solving (10 points each, 30 points total)

11. A rotating shaft is subjected to a fully reversed bending moment of 500 N-m. The shaft is made of a material with an ultimate tensile strength of 700 MPa, a yield strength of 350 MPa, and a fully corrected endurance limit of 300 MPa. The shaft has a diameter of 30 mm, a minor diameter of 28.5 mm, and a circumferential groove with a radius of 1.5 mm (D = 30 mm, d = 28.5 mm, r = 1.5 mm), as pictured in the figure below. Determine the life of the component. Show all calculations.

Figure A–15–14

Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$D/d = 30/28.5 = 1.05$

$r/d = 1.5/28.5 = 0.05$

$K_t = 2.1$

S_e GIVEN, SO WE CAN SKIP TO STEP 3 IN ROAD MAP

3. $K_f = 1 + \dfrac{K_t - 1}{1 + \sqrt{a/r}}$

BENDING: $\sqrt{a} = 1.24 - 2.25 \times 10^{-3} S_{ut} + 1.60 \times 10^{-6} S_{ut}^2 + 4.11 \times 10^{-10} S_{ut}^3$

$= 0.308$

$\sqrt{r} = \sqrt{1.5} = 1.22$

$\sqrt{a/r} = \dfrac{0.308}{1.22} = 0.25$

$K_f = 1 + \dfrac{2.1 - 1}{1 + 0.25} = 1.88$

FULLY REVERSED LOAD, SO $\sigma_m = 0$

$$\sigma_{ao} = \frac{Mc}{I} \qquad c = \frac{d}{2} = \frac{28.5\text{ mm}}{2} = 14.25\text{ mm}$$

CIRCULAR CROSS-SECTION $\quad I = \frac{\pi d^4}{64} = \frac{\pi(0.0285\text{ m})^4}{64} = 3.24\times10^{-8}\text{ m}^4$

$$\sigma_{ao} = \frac{(500\text{ N}\cdot\text{m})(0.01425\text{ m})}{3.24\times10^{-8}\text{ m}^4} = 220\text{ MPa}$$

4. $\sigma_a = K_f\,\sigma_{ao}$

$$= 1.88\,(220\text{ MPa})$$

$$= 413\text{ MPa}$$

5. $f = 1.06 - 4.1\times10^{-4}S_{ut} + 1.5\times10^{-7}S_{ut}^2$

$$= 0.85$$

$$a = \frac{(fS_{ut})^2}{S_e} = 1170$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -0.099$$

$$N = \left(\frac{\sigma_a}{a}\right)^{\frac{1}{b}} = 38{,}600 \text{ CYCLES TO FAILURE}$$

<u>BONUS</u>: DON'T FORGET TO CHECK FOR YIELD!

$$n_y = \frac{S_y}{\sigma_m + \sigma_a} = \frac{400}{413} = 0.97$$

YIELD PREDICTED AT NOTCH ON 1st CYCLE!

12. A solid steel shaft transmits 20 kW of power at 1200 rpm. Torque is transmitted to the shaft through a key with dimensions: width = 8 mm, depth = 5 mm, length = 15 mm. The key material has a yield strength S_y = 310 MPa and ultimate tensile strength S_{ut} = 400 MPa.
    a. Calculate the minimum shaft diameter to prevent key shearing with a safety factor of 1.5.
    b. Determine the shaft diameter needed to avoid key crushing with the same safety factor.

(*Hint:* To convert power (P) in kW to torque (T) in N-m, you can use the formula:

$$T = \frac{9550 \cdot P}{N}$$

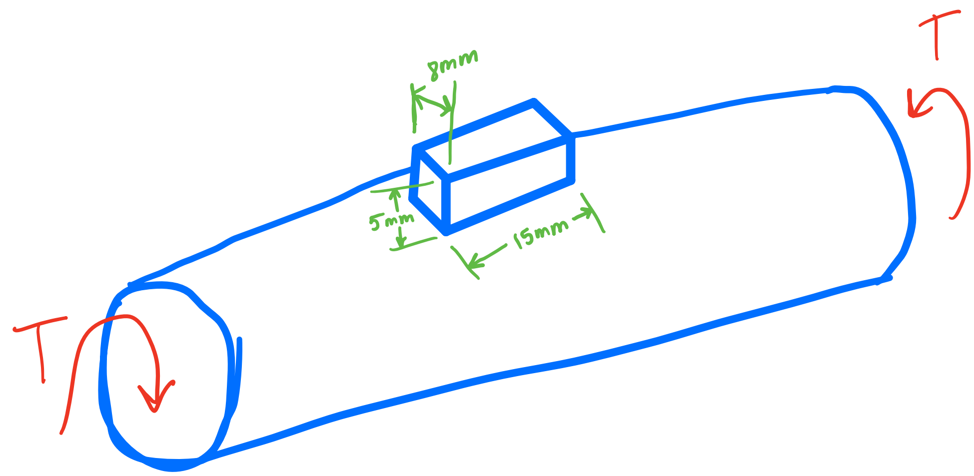
where *N* is the speed in rpm.)

$P = 20\ \text{kW}$

$N = 1200\ \frac{\text{rev}}{\text{min}}$

$S_y = 310\ \text{MPa}$

$S_{ut} = 400\ \text{MPa}$



$$T = \frac{9550(20\ \text{kW})}{1200\ \frac{\text{rev}}{\text{min}}} = 159\ \text{N}\cdot\text{m}$$

(Note that unit conversions are contained in the 9550 factor here)

(a) $n = 1.5 = \dfrac{0.577 S_y}{\tau}$

$$\Rightarrow \tau = \frac{0.577(310\ \text{MPa})}{1.5} = 119\ \text{MPa}$$

$$\tau = \frac{T/r}{w \cdot l} \Rightarrow r = \frac{T/\tau}{w \cdot l} = \frac{(159\ \text{N}\cdot\text{m})/(119 \cdot 10^6\ \text{Pa})}{(0.008\ \text{m})(0.015\ \text{m})}$$

$$= 0.011\ \text{m}$$

$$= 11\ \text{mm}$$

$$\boxed{d_{min} = 22\ \text{mm}}$$

(b) $n = 1.5$

assume $S_{yc} = S_{yt} = S_y$

$$n = \frac{S_{yc}}{\sigma} = \frac{S_y}{\sigma}$$

$$\Rightarrow \sigma = \frac{310\text{ MPa}}{1.5} = 207\text{ MPa}$$

$$\sigma = \frac{T/r}{\frac{h}{2} \cdot l} \Rightarrow r = \frac{T/\sigma}{\frac{h}{2} \cdot l}$$

$$r = \frac{(159\text{ N}\cdot\text{m})/(207\times10^{6}\text{ Pa})}{\left(\frac{0.005}{2}\text{ m}\right)(0.015\text{ m})}$$

$$= 0.020\text{ m}$$

$$= 20\text{ mm}$$

$$\boxed{d_{min} = 40\text{ mm}}$$

13. A compression spring in a mechanical assembly is subjected to cyclic loading, varying from 80 N to 400 N. The spring material is chrome-vanadium wire with a diameter of 2.5 mm and the spring is peened. The outer diameter of the spring coil is 25 mm. FInd:
    a. The endurance limit of the spring wire, Sse, using the Goodman criterion with Zimmerli data.
    b. The midrange (τ_m) and alternating (τ_a) components of shear stress for the spring.
    c. The factor of safety for infinite life using the Goodman failure criterion.

Table 10-4: Constants for Estimating Minimum Tensile Strength of Common Spring Wires

Material	ASTM No.	Exponent *m*	Diameter, in	*A*, kpsi · in^m	Diameter, mm	A, Mpa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004 to 0.256	201	0.10 to 6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020 to 0.500	147	0.5 to 12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028 to 0.500	140	0.7 to 12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032 to 0.437	169	0.8 to 11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063 to 0.375	202	1.6 to 9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013 to 0.10	169	0.3 to 2.5	1867	7.6 to 11
		0.263	0.10 to 0.20	128	2.5 to 5	2065	
		0.478	0.20 to 0.40	90	5 to 10	2911	
Phosphor-bronze wire**	B159	0	0.004 to 0.022	145	0.1 to 0.6	1000	8.0
		0.028	0.022 to 0.075	121	0.6 to 2	913	
		0.064	0.075 to 0.30	110	2 to 7.5	932	

$F_{min} = 80\,N$

$F_{max} = 400\,N$

$d = 2.5\,mm$

$D_o = 25\,mm$

(a) $$S_{se} = \frac{S_{sa}}{1 - \frac{S_{sm}}{S_{su}}}$$

Zimmerli data: $S_{sa} = 398\,MPa$, $S_{sm} = 534\,MPa$

$$S_{su} = 0.67\,S_{ut}$$

$$S_{ut} = \frac{A}{d^m} = \frac{2005\ MPa\cdot mm^m}{(2.5\ mm)^{0.168}} = 1719\ MPa$$

$$S_{su} = 0.67(1719\ MPa) = 1152\ MPa$$

$$S_{se} = \frac{398\ \text{MPa}}{1 - \frac{534\ \text{MPa}}{1152\ \text{MPa}}}$$

$$\boxed{S_{se} = 742\ \text{MPa}}$$

(b) $\tau_a = K_B \dfrac{8F_a D}{\pi d^3}$, $\tau_m = K_B \dfrac{8F_m D}{\pi d^3}$

$$K_B = \frac{4C+2}{4C-3}$$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{400\text{N} - 80\text{N}}{2}$$

$$F_a = 160\text{N}$$

$$C = \frac{D}{d}$$

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{400 + 80\text{N}}{2}$$

$$= 240\text{N}$$

$$D = D_o - d$$

$$= 25\text{mm} - 2.5\text{mm} = 22.5\text{ mm}$$

$$C = \frac{22.5\text{ mm}}{2.5\text{mm}} = 9.0$$

$$K_B = \frac{4(9)+2}{4(9)-3} = 1.15$$

$$\tau_a = (1.15)\frac{8(160\text{N})(0.0225\text{m})}{\pi(0.0025\text{m})^3}$$

$$= 6.75 \times 10^8\ \text{Pa}$$

$$\boxed{\tau_a = 675\ \text{MPa}}$$

$$\tau_m = (1.15)\frac{8(240\,N)(0.0225\,m)}{\pi(0.0025\,m)^3}$$

$$= 1.01 \times 10^9\ Pa$$

$$\boxed{\tau_m = 1012\ MPa}$$

(c) $$\frac{1}{n_f} = \frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}$$

$$\frac{1}{n_f} = \frac{675\ MPa}{742\ MPa} + \frac{1012\ MPa}{1152\ MPa}$$

$$\Rightarrow \boxed{n_f = 0.60}$$

∞ LIFE NOT PREDICTED